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Chaos Theory

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Abstract

Chaos theory is the study of dynamic systems that are highly sensitive to initial conditions. Although there are many systems that are chaotic, like the weather or a double pendulum, these systems are often connected through universality, a shared property of chaotic systems that is independent of the specific dynamics. In this study of chaos, we analyze a chaotic system, a Resistor-Inductor-Diode (RLD) circuit, qualitatively and quantitatively, by studying system characteristics such as period doubling, chaotic bandwidth, and the system's forms of universality, notably the Feigenbaum ratio. These properties, although studied in the specific case of the RLD circuit, can be used to make claims about chaotic systems in general.

1. Introduction

The simplest way to characterize chaotic behavior is in a system's extreme sensitivity to initial conditions. The weather is good example of a commonplace chaotic system. Edward Lorenz originally tried to describe atmospheric convection using a set of ordinary differential equations called the Lorenz equations. He quickly found that it was impossible to make predictions about the system. What he initially thought were insignificant roundings in his numerical calculations lead him to hypothesize the chaotic nature of the weather. He summarized chaos in the following way: ”When the present determines the future, but the approximate present does not determine the approximate future” [2]. Systems that are deterministic can also chaotic. A system that is chaotic is deterministic if its future can be predicted if one has perfect knowledge of the initial conditions. It is important to distinguish this kind of behavior from a random system, in which predictions cannot be made about the future, even with perfect knowledge of initial conditions. In general, the difficulty in predicting chaotic systems arise because it is impossible to have perfect knowledge of initial conditions. As seen in the example with the weather, even the smallest approximation of a chaotic system will lead to a diverging solution. This paper presents a qualitative and quantitative look at a nonlinear and chaotic system, an RLD circuit. We will visualize chaotic behavior in three different ways, study the relationship between the driving voltage and period doubling, approximate the Feigenbaum ratio, compare experimental and theoretical predictions of universality constant τ through the chaotic bandwidth, produce bifurcation plots, and simulate an RLD circuit using Mathematica.

2. Theory

2.1. Chaos

An illuminating example of chaos can be seen through the Lorenz system:

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

where x, y, and z make up the system state, t is time, and σ, ρ, and β, are the system parameters. What appear to be a simple set of first order differential equations leads to a set of solutions that are chaotic and that have great detail and complexity. The phase space plot of the solutions of the Lorenz system is an example of a strange 3 attractor [5], which is a set of numerical values towards which a system tends to evolve, for a wide variety of starting conditions, and is considered strange is if it has a fractal structure [1]. If a strange attractor is chaotic, then any two arbitrarily close alternative initial points will lead to points that are both arbitrarily far apart and arbitrarily close.

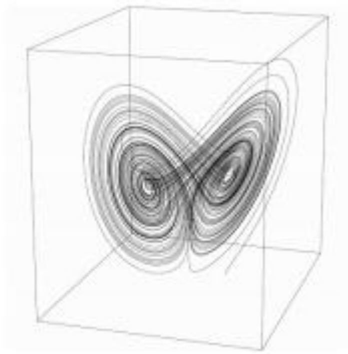

Figure 1: Lorenz Attractor [3]

Although we will not be studying the Lorenz attractor, it illustrates the rich dynamics of chaos through a set of simple first order differential equations. To understand chaos further, we will delve deeper into the example of an RLD circuit.

2.2 Period Doubling

Period doubling is a common characteristic of the progression of a chaotic system as the system tends towards chaos. For such systems, as one increases the driving parameter, a new behavior of the system occurs with twice the period of the original behavior of the system. A period doubling cascade is sequence of period doublings and further doublings as the driving parameter is increased further and further. In the specific case of an RLD circuit, as we change the driving voltage, the system experiences period doubling on its way towards chaos. In our experiment, period doubling and its representation using a bifurcation map will be observed in the RLD circuit.

2.3 Universality

Universality is the property of a class of systems that exhibits similar behavior independent of dynamical details. This property, usually a constant, connects many chaotic systems together. For systems that approach chaos via period doubling, like the RLD circuit, one example of a universality is the Feigenbaum ratio:

$$\delta = \lim_{n\to\infty} \frac{\gamma_n - \gamma_{n-1}}{\gamma_{n+1} - \gamma_n}$$

where γn is driving parameter at the nth bifurcation. The Feigenbaum ratio describes the ratio between the difference of the driving parameter at the nth and n−1th bifurcation and the difference of the driving parameter at the n+1th and nth bifurcation. The Feigenbaum ratio connects systems or functions that approach chaos via period doubling and show an underlying connection between period doubling chaotic systems. Another form of universality constant τ can be determined through the chaotic bandwidth, the width of the range of the solution to a chaotic system while the system is in chaos.

T ~ (γ − γchaos) τ

where γ chaos is the driving parameter that the system first reaches chaos, γ is the current driving parameter while in chaos, and τ is the universality constant.

3. Experimental Setup

Our setup is a Resistor-Inductor-Diode (RLD) circuit. On a high level, the input of the system is a driving voltage VDrive and frequency f and the output is an output voltage Vout. The experiments involve a combination of holding f constant while varying VDrive and holding VDrive constant while varying f.

3.1. The RLD Circuit

The main components of the circuit include a resistor, an inductor, and a diode. The function generator outputs a driving voltage and frequency that is passed through the first oscilloscope channel and through the RLD circuit. The diode is the nonlinear component, and after the voltage passes through the diode and the inductor, the output voltage is recorded in the second oscilloscope channel. The remainder of the circuit is the resistor and the ground. Fig. 2 shows a detailed circuit schematic and its output/input connections.

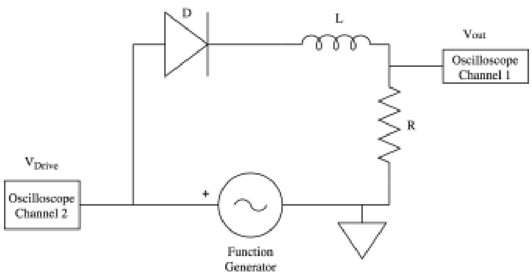

Figure 2: RLD Circuit

3.2. Full Experimental Setup

The full setup is a more detailed experimental schematic. Notable details include an amplifier to amplify the output voltage and the specific digital oscilloscope Picoscope. Picoscope's benefits include computer software that allows us to record and analyze data on the computer. Not included in the schematic is a noise reduction device connected to the circuit. Fig. 3 shows a detailed full setup schematic and its output/input connections.

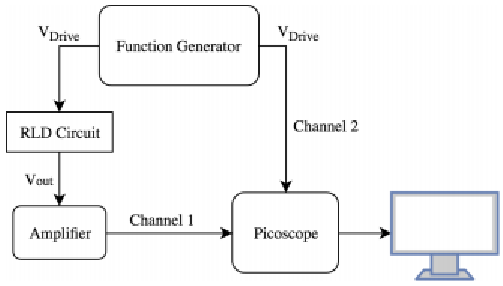

Figure 3: Full experimental setup

4. Data Collection and Results

Data is collected to understand the chaotic behavior of the RLD circuit, which we represent qualitatively and quantitatively. The first qualitative experiment of the chaotic behavior, specifically the phenomenon of period doubling, is through three different visualizations of the data: a voltage over time, phase space, and a power spectrum. The remainder of the experiments include quantitative measures of chaos through plotting the occurrences of bifurcation and chaos, calculating the Feigenbaum ratio and τ , measuring chaotic bandwidth, and generating a bifurcation map.

4.1. Visualizing Chaotic Behaviors in an RLD Circuit

As a reminder, the inputs of our system are a driving voltage and frequency. We collected data between the frequencies of 35 - 175kHz for voltages from 1 - 20V. For each frequency starting at 35kHz, we increase the driving voltage starting from 1V and record the driving voltage where the system's behavior changes to a new behavior or returns to an old behavior. There are three changes in behavior that we observe: 1) The period doubles. Ex. period goes from T to 2T; 2) the behavior of the system reverts back to a previous behavior. Ex. the period is 4T and returns to 2T; 3) the system enters or exits chaos. Ex: T is not well defined.

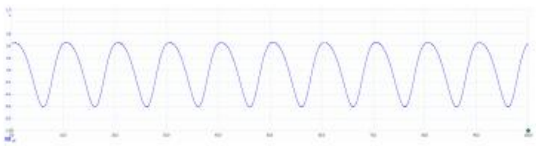

Figure 4: Voltage over time. Period T

Figure 5: Phase space. Period T

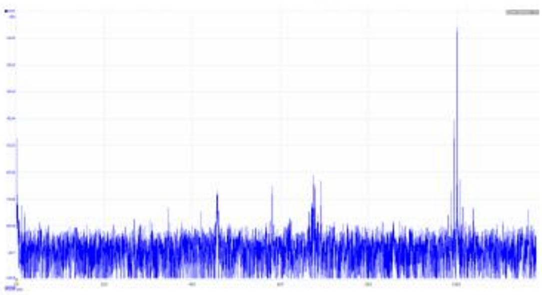

Figure 6: Power spectrum. Period T

In Fig. 4-6, three different representation are used to visualize the same frequency and dynamics of the system at period T. In each of the following pages, each cluster of three representations has the same driving voltage. Fig. 4 is voltage over time with time on the x-axis and output voltage on the y-axis, Fig. 5 is a phase space diagram with driving voltage on the x-axis and the driving voltage of the y-axis, and Fig. 5 is a power spectrum with frequency on the x-axis and the decibels on the y-axis. There is period of T = 1 f in the voltage over time plot, one period ring in phase space, and one peak on the power spectrum at 100kHz, the initial driving frequency. This is the behavior of the system until we reach the first bifurcation. With the frequency set at 100kHz, we begin to vary the driving voltage from 1 to 20V.

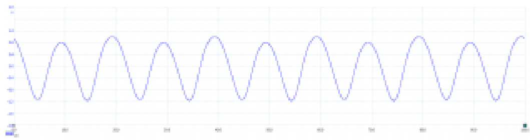

Figure 7: Voltage over time. Period 2T

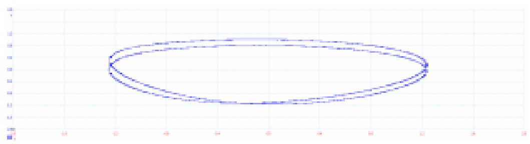

Figure 8: Phase space. Period 2T

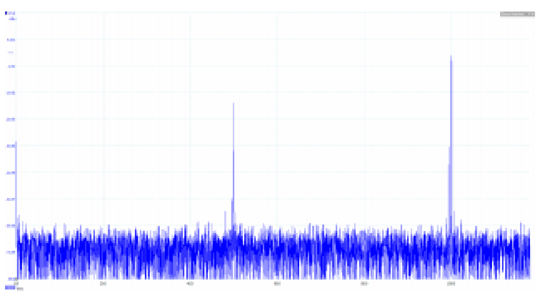

Figure 9: Power spectrum. Period 2T

As the driving voltage is increased, the first change in behavior occurs at around 2.5V. The period of the system double from T to 2T, which we call one bifurcation. The way we can interpret these plots now is that the period has doubled from T to 2T, corresponding to two different peak output voltages in the voltage time plot. The corresponding observation in the phase space plot is the two period rings, which means the system is oscillating between two output voltages at each driving voltage. From the power spectrum, there are two peaks, one at 100kHz and one at 50kHz, corresponding to the original driving frequency and the halved frequency (T = 1 f or doubled period). The Fig. 7-9 show the behavior of the system when the system has bifurcated and the period is 2T.

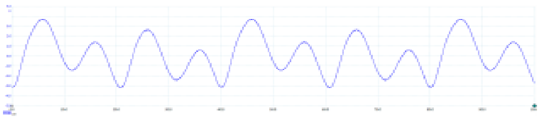

Figure 10: Voltage over time. Period 4T

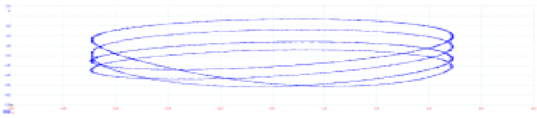

Figure 11: Phase space. Period 4T

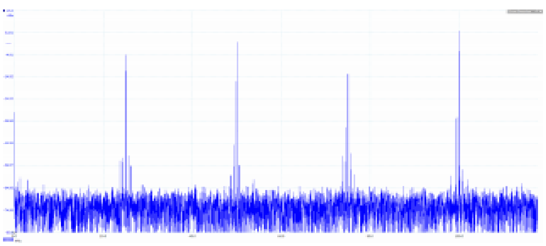

Figure 12: Power spectrum. Period 4T

At around 6.7V, another bifurcation occurs and the period doubles again from 2T to 4T. The period is now 4T or four times the original period of T or double the period of 2T. We can confirm visually that this by inspecting the plots once again. There are four output voltage peaks in the voltage over time, four period rings in the phase space plots, and four peaks, corresponding to 25kHz, 50kHz, 75kHz, and 100kHz in the power spectrum plot. We continue increasing the driving voltage until the next behavior change.

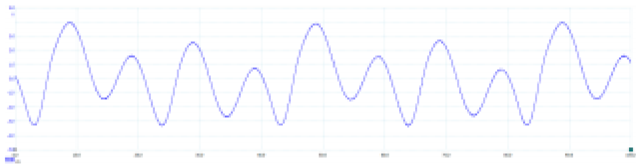

Figure 13: Voltage over time. Period 8T

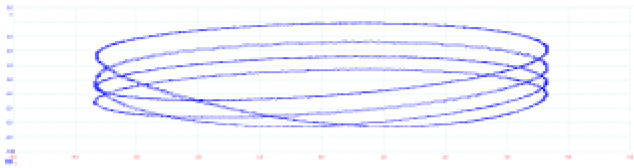

Figure 14: Phase space. Period 8T

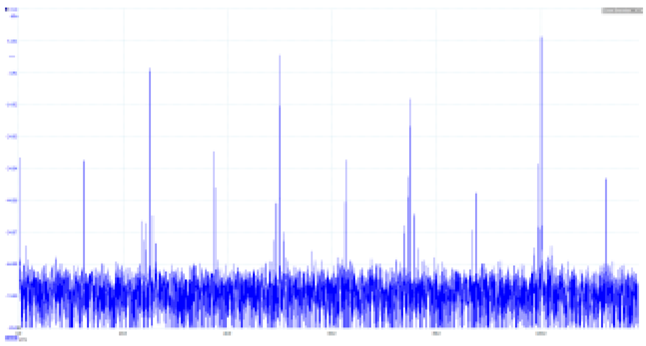

Figure 15: Power spectrum. Period 8T

We continue forward and at around 7.3V, the next bifurcation occurs and the period doubles again from 4T to 8T. We inspect this in the same way that we did the past two period doublings. Although this period doubling is not as obvious in Fig. 13 and 14, it is visually confirmed by the power spectrum that there are eight frequency spikes, corresponding to eight different peak output voltages. For the sake of not including every period doubling, we do not include visualizations of the the period doubling from 8T to 16T. Instead, the next cluster of plots show the system once it has reached chaos.

At around 8.4V, we have finally reached chaos. We can visually inspect this by seeing that the voltage over time plot has no repeating peaks, the phase space has many period rings across a range of output voltages, and the power spectra has a broad range of frequencies with no particular standouts (outside of the driving frequency of 100kHz). Through period doubling, the system has reached chaotic behavior. Interestingly, the system eventually exits chaos and enters a new behavior with a period of 3T.

Figure 16: Voltage over time. Chaos

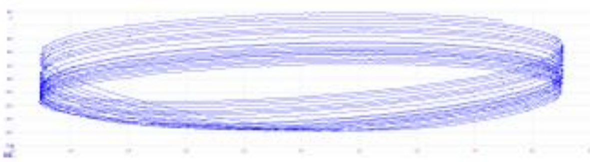

Figure 17: Phase space. Chaos

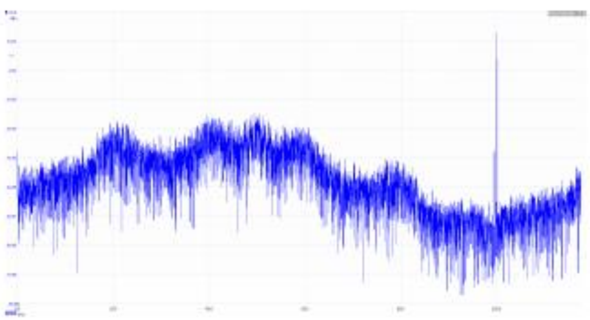

Figure 18: Power spectrum. Chaos

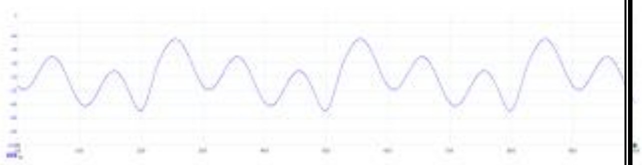

Figure 19: Voltage over time. Period 3T

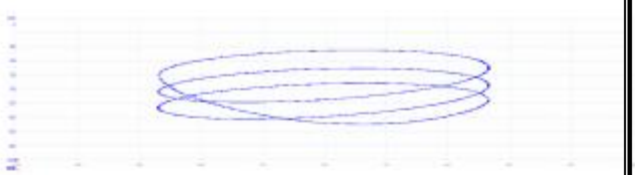

Figure 20: Phase space. Period 3T

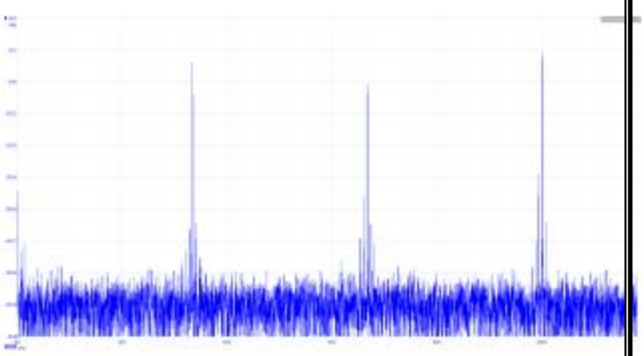

Figure 21: Power spectrum. Period 3T

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Continuing forward, we exit chaos at around 10V, and hit a system with a period of 3T. We can inspect the plots and notice the three peaks, the three period rings in the phase space, and the power spectra with three peaks at around 33.3kHz, 66.6 kHz, and 100kHz. Continuing to increase the driving voltage, the behavior of the system changes to a system with a period of 6T and then 12T. The full story and occurrences of period doublings and chaos is fully shown in the next section.

4.2. Occurrences of Bifurcation and Chaos

The occurrences of bifurcation (period doubling) and chaos are more complex than starting at a period and continuously doubling until chaos is reached. The figure below shows the full story.

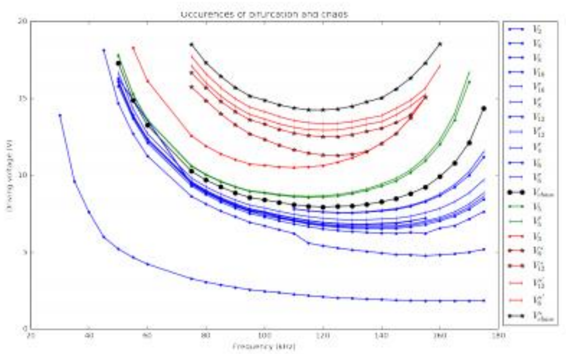

Figure 22: Occurrences of bifurcation and chaos

In this plot, we show the full progression towards chaos through the occurrences of bifurcations and chaos in a system. The x-axis is frequency and the y-axis is the driving voltage. Starting from the bottom, we interpret the graph by travelling up the graph. Each line represents a new behavior, either a bifurcation or chaos. The lines in blue are the bifurcations that occur before the first chaos and the red ones are the bifurcations that occur after the first chaos but before the second one. The green lines are an anomaly that we found that occurred during some of the frequencies in which we would see a period of 5T. Black lines represent when the system is in chaos. The special notation we have here include solid points, primes, and stars. A solid point represents a new bifurcation (ex. from 2T to 4T), a vertical line on a data point represents a period disappearing (ex. from 8T to 4T), and a star on the point represents that same period showing up a second time (ex. 4T appearing a second time). The white space in between each line is the range of driving voltages that the system's period is the subscript number multiplied by T. For example, the white space between V2 and V4 is the range of driving voltages that the system has a period of 2T. The full data set is found in the next few pages.

The following tables below show the full data we collected. For consistency, we always take data while increasing the driving voltage, never decreasing. We do this because the voltage where a behavior change happens changes depending on the direction of approach. Frequencies have error ±0.1kHz and voltages have error ±0.05V, which we get through estimation.

f(kHz)	V_2	V_4	V_8	V_{16}	V_{16}'	V_8'	V_{12}
30	13.90						
35	9.61						
40	7.60						
45	5.99	18.14					
50	5.21	14.68	15.80	16.29	16.30	16.02	16.10
55	4.66	12.71	13.70			13.81	13.91
60	4.22	11.25	12.10			12.26	12.35
65							
70							
75	3.27	8.63	9.31	9.41	9.45	9.48	9.55
80	3.06	8.13	8.74	8.84	8.89	8.93	9.00
85	2.87	7.67	8.26	8.38	8.42	8.46	8.53
90	2.71	7.30	7.86	7.97	8.02	8.06	8.13
95	2.56	6.94	7.51	7.64	7.68	7.72	7.80
100	2.46	6.71	7.29	7.39	7.49	7.58	7.60
105	2.37	6.45	7.04	7.19	7.24	7.28	7.37
110	2.26	6.22	6.83	7.02	7.05	7.09	7.18
115	2.17	5.59	6.66	6.83	6.88	6.93	7.04
120	2.10	5.41	6.51	6.70	6.77	6.81	6.92
125	2.02	5.27	6.40	6.61	6.68	6.73	6.84
130	2.00	5.14	6.33	6.55	6.63	6.69	6.80
135	1.95	5.05	6.28	6.53	6.60	6.68	6.80
140	1.92	4.94	6.24	6.52	6.63	6.69	6.83
145	1.87	4.85	6.22	6.54	6.64	6.71	6.86
150	1.86	4.82	6.29	6.64	6.73	6.82	7.01
155	1.84	4.77	6.22	6.79	6.92	7.02	7.21
160	1.84	4.82	6.56	7.03	7.14	7.25	
165	1.84	4.89	6.72	7.34	7.45	7.61	
170	1.84	5.00	7.15	7.79	7.93	8.12	
175	1.86	5.18	7.64	8.43	8.62	8.84	

Table 1: Driving voltages at which bifurcations and chaos occurs for driving frequency from 30kHz to 175 kHz (Part 1 of 3)

f(kHz)	V_{12}'	V_4'	V_6	V_6'	V_{chaos}	V_5	V_5'
30							
35							
40							
45							
50	16.18	16.67			17.30	17.79	17.82
55	13.93	14.34			14.87	15.27	15.30
60	12.37	12.70			13.25	13.58	13.60
65							
70							
75	9.59	9.82			10.26	10.58	10.62
80	9.02	9.25			9.69	10.02	10.06
85	8.55	8.78			9.24	9.58	9.62
90	8.16	8.39			8.86	9.23	9.27
95	7.82	8.05			8.55	8.95	8.98
100	7.63	7.87			8.40	8.84	8.89
105	7.41	7.65			8.23	8.69	8.75
110	7.22	7.46	7.81	7.82	8.09	8.60	8.67
115	7.07	7.31	7.69	7.70	7.99	8.56	8.62
120	6.94	7.20	7.60	7.62	7.94	8.58	8.66
125	6.87	7.13	7.56	7.58	7.98	8.65	8.75
130	6.83	7.09	7.55	7.58	8.00	8.80	8.89
135	6.83	7.11	7.60	7.63	8.11	9.01	9.11
140	6.87	7.15	7.67	7.72	8.27	9.29	9.42
145	6.91	7.20	7.77	7.83	8.48	9.63	9.79
150	7.03	7.33	7.98	8.04	8.81	10.18	10.34
155	7.21	7.56	8.27	8.33	9.23	10.93	11.16
160		7.84	8.67	8.77	9.91	12.00	12.28
165		8.27	9.22	9.38	10.79	13.59	13.99
170		8.85	9.99	10.23	12.12	16.08	16.68
175		9.70	11.18	11.53	14.36		

Table 2: Driving voltages at which bifurcations and chaos occurs for driving frequency from 30 kHz to 175 kHz (Part 2 of 3)

f(kHz)	V_3	V_6^*	V_{12}^*	$V_{12}^{*'}$	$V_6^{*'}$	V_{chaos}^*
30						
35						
40						
45						
50						
55	18.29					
60	16.13					
65						
70						
75	12.57	15.75	16.66	17.12	17.73	18.52
80	11.90	14.86	15.69	16.01	16.60	17.31
85	11.39	14.00	14.81	15.15	15.69	16.44
90	11.02	13.28	14.14	14.48	14.98	15.70
95	10.72	12.69	13.60	13.95	14.42	15.15
100	10.65	12.33	13.32	13.65	14.13	14.88
105	10.54	11.95	12.98	13.34	13.81	14.57
110	10.49	11.66	12.74	13.11	13.56	14.38
115	10.54	11.43	12.58	12.97	13.41	14.24
120	10.63	11.31	12.50	12.93	13.35	14.24
125	10.84	11.30	12.52	12.95	13.37	14.31
130	11.12	11.36	12.63	13.09	13.48	14.48
135	11.53	11.54	12.85	13.30	13.69	14.77
140	12.07	12.07	13.05	13.47	13.87	15.02
145	12.70	12.70	13.42	13.92	14.34	15.60
150	13.73	13.73	13.92	14.50	14.87	16.32
155	15.07	15.07	15.07	15.24	15.65	17.30
160					17.11	18.54
165						
170						
175						

Table 3: Driving voltages at which bifurcations and chaos occurs for driving frequency from 30 kHz to 175 kHz (Part 3 of 3)

In general, the occurrences of bifurcations and chaos are complex and dynamically rich, and not easy to understand. It is not obvious to us, for example, why we observed a period of 5T for some frequencies in the middle of chaos, or why the system exits chaos and then begins to behave like a system with a period of 3T. The visualizations show the RLD circuits path to chaos through period doubling, but also reveals many unanswered questions about the nature of chaotic systems.

4.3. Feigenbaum Ratios

Universality is the property of a class of systems that exhibits similar behavior independent of dynamical details and is what connects many chaotic systems together. The Feigenbaum ratio δ, a universal constant for functions approaching chaos via period doubling[4], is one such example of universality. The equation is repeated from earlier for convenience.

$$\delta = \lim_{n\to\infty} \frac{\gamma_n - \gamma_{n-1}}{\gamma_{n+1} - \gamma_n}$$

In the case of the RLD circuit, the Feigenbaum ratio can be approxi from our collected data using

$$f = \delta \approx \frac{V_8 - V_4}{V_{16} - V_8}$$

where V8 is the bifurcation at which the period is 8T, V4 is the bifurcation at which the period is 4T, and V16 is the bifurcation at which the period is 16T. We approximate the Feigenbaum ratio for the most consistent part of our data between frequencies 110 - 150 kHz. With an estimated error of 0.05V, we propagate the error and find a value σ away from the true value.

f(kHz)	Calculated δ	Error δ	σ
110	3.211	1.394	1.05
115	6.294	2.834	0.58
120	5.789	2.362	0.48
125	5.380	1.988	0.36
130	5.409	1.907	0.39
135	4.920	1.540	0.17
140	4.643	1.306	0.03
145	4.281	1.063	0.37
150	4.200	0.956	0.50

Table 4: Approximate Feigenbaum ratios taken using bifurcations at 4T, 8T and 16T.

In the literature, the accepted value of the Feigenbaum ratio is δ = 4.6692016. Taking the average across our calculated Feigenbaum ratios and errors, we can get an estimate for the Feigenbaum ratio:

$$\delta = 4.903 \pm 1.706$$

The formula we used to propagate error is the following.

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial V_8}\delta V_8\right)^2 + \left(\frac{\partial f}{\partial V_4}\delta V_4\right)^2 + \left(\frac{\partial f}{\partial V_{16}}\delta V_{16}\right)^2}$$

$$\delta f = \sqrt{\left(\frac{(V_8 - V_4) + (V_{16} - V_8)}{(V_{16})^2}\delta V_8\right)^2 - (\delta V_4)^2 + \left(\frac{-(V_8 - V_4)}{(V_{16} - V_8)^2}\delta V_{16}\right)^2}$$

Our approximation is within 0.14σ of the true value, so we can conclude that this particular part of the data is very good. The Feigenbaum ratio that we calculated was only for a small portion of the data, and although the calculation is quite precise, we expect that the Feigenbaum ratio will have a higher variance for other parts of the data. Given more time, we could calculate the Feigenbaum ratio over the frequency range 30 - 175kHz for each possible bifurcation, giving us a more accurate measure of the quality of our data. However, for the small part of the data that we considered, the approximation of the Feigenbaum value that we calculated is promising for future calculations.

4.4. Chaotic Bandwidths

In phase space, chaos is bound by the chaotic bandwidth, the length of the range of the output voltage for a given driving voltage. The chaotic bandwidth is the relationship between the voltage that the system first enters chaos and the range of driving voltages while the system is in chaos.

$$T \sim (V - V_{chaos})^{\tau}$$

where Vchaos is the driving voltage that chaos first ocurs, V is the current driving voltage, and τ is the universality constant. Using Picoscope, we measure the chaotic bandwidth with a digital measuring tool. Fig. 23 shows the chaotic bandwidth T for a system at 100kHz in chaos. We measure the chaotic bandwidth ten times for each driving voltage and take the average, stopping the Picoscope at random times in order to reduce systematic error. For a given frequency, we find the driving voltage Vchaos where chaos first begins and increase the driving voltage in increments of 0.1V and calculate ΔV , the difference between Vchaos and the current driving voltage. We take the measurement at 120kHz and 140kHz and calculate τ with some constant of proportionality.The x-axis in Fig. 24 is ΔV in Volts and the y-axis is the measured chaotic bandwidth in millivolts with error bars calculated using the standard deviation computed across the ten trials for each chaotic bandwidth.

Using a scipy curve fitting package, we calculate the following universality constants:

$$\tau_{120} = 0.0174 \pm 0.004$$
$$\tau_{140} = 0.0236 \pm 0.008$$

where τ120 is the universality constant measured for 120kHz and τ140 is the universality constant measured for 140kHz. Both universality constants fall 18 within 1-2σ of each other, and we can reason that two systems with different driving frequencies share a universality constant τ . The behavior of the RLD circuit is connected not only through the well known Feigenbaum ratio δ but also the universality constant τ .

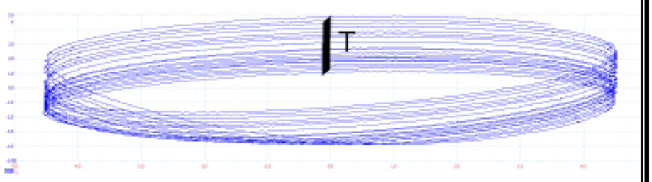

Figure 23: Chaotic bandwidth marked with T

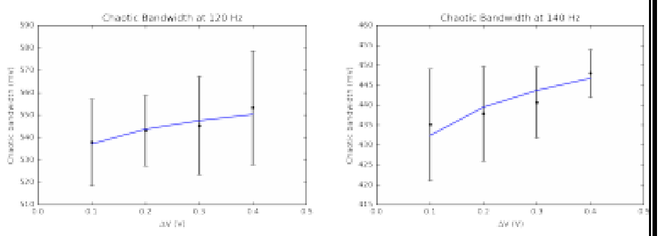

Figure 24: Left: Chaotic bandwidth at 120 kHz. Right: Chaotic bandwidth at 140 kHz

4.5. Bifurcation Maps

A common way to visualize period doubling is through a bifurcation map, a plot of bifurcations as a function of the driving parameter. The x-axis is the driving voltage and the y-axis is output voltage of the voltage peaks. For example, in the range between 2V and 7V of the driving voltage, the system is oscillating between two output voltage peaks. This signals that we are in the region of period 2T. This particular region from 2 - 7V can be interpreted as a fourth representation of the 2T system seen in Fig. 7-9. As the driving voltage continues to increase, the period doubles again and again until we enter the first region of chaos, marked by the first white space between 8-11V, exits chaos, enters a behavior with period 3T, and then returns chaos after about 14V, marked by the second white space from 14 - 20V.

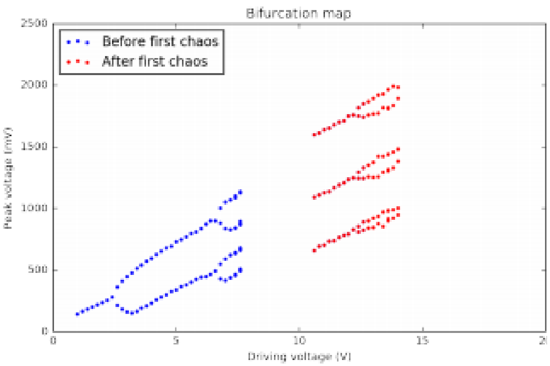

Figure 25: Bifurcation maps at 100 Hz from 1 - 20V at increments of 0.20 V through the first and second chaos. The white region between 8V and 11V is the first region of chaos and the white region from 14-20V is the second region of chaos.

4.6. Simulation

Using Mathematica, we can simulate an RLD circuit by simulating a circuit with two different capacitors:

$$L\frac{dI}{dt} = -RI(t) - \left(\frac{C_2 - C_1}{2C_1C_2}|q(t)| + \frac{C_1 + C_2}{2C_1C_2}q(t) + E_0\right) + V_0 sin(2\pi f t)$$

where I is the current, L is the inductance, C1 is the capacitance of the first capacitor, C2 is the capacitance of the second capacitor, q(t) is the charge, E0 is an off set voltage, and V0 is the driving voltage.

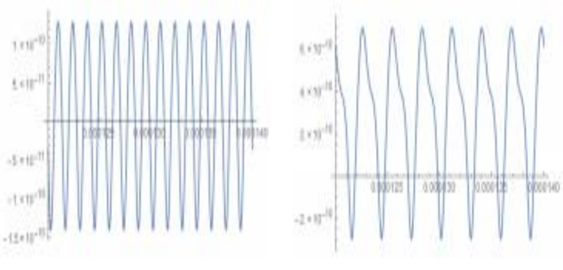

Figure 26: Charge over time at $V_{Drive} = 0.1$ (left) and 0.2 (right)

From these simulations, the x-axis is the charge and the y-axis is time. These plots are show the same period doubling as the voltage over time plots generated.

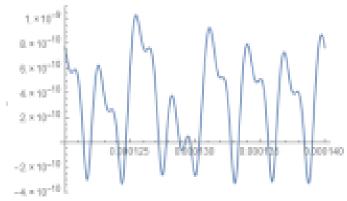

Figure 27: Charge over time at $V_{Drive} = 0.3$

with our collected data. At Vdrive = 0.1, the period is T. At Vdrive = 0.2, the has doubled to 2T. At Vdrive = 0.3, the system is in chaos and the period cannot be determined. Other plots can be generated from the Mathematica simulation, like phase space plots and bifurcation plots, but due to time restrictions, we were only able to produce plots for charge over time. In the future, it would be instructive to simulate the experiment on Mathematica and compare the results with collected data.

5. Summary

We set the goal of understanding chaotic behavior in RLD circuits. First, we visualized chaotic behavior in an RLD circuit in three different representations: voltage over time, phase space, and power spectrum. These different representations allowed us to get a qualitative understanding of period doubling and the occurrences of bifurcations as the RLD circuit progressed towards chaos. Next, we discussed universality and how chaotic systems are connected through universality constants. The Feigenbaum ratio was an example of such a universality constant, which we approximated over a range of frequencies and driving voltages where the system had period 4T, 8T, and 16T. The value we calculated was found to be well within the accepted value of $\delta = 4.6692016$.

$$\delta = 4.903 \pm 1.706$$

Next, we compared measurements of chaotic bandwidth and the theoretical predictions to compute the universality constant τ for a driving frequency of 120kHz and 140kHz. We found that the two values of τ were within 1-2σ of each other and therefore it was reasonable to claim that τ was another example of a universality constant in an RLD circuit.

$$\tau_{120} = 0.0174 \pm 0.004$$
$$\tau_{140} = 0.0236 \pm 0.008$$

Next, we created a bifurcation map of our data showing the progression of the RLD circuit towards its first and second chaos through period doubling. We 21 noted that the bifurcation diagram is another way of visualizing the behavior of a chaotic system, like we did in the first part of the experiment. Lastly, we simulated an RLD circuit in Mathematica showing charge over time plots for different driving voltages, which helped confirm the validity of the voltage over time plots generated using the data that we collected. Overall, we gained a lot of insight both into the nature of chaos in an RLD circuit and chaos in general.

6. Conclusion

RLD circuits are a good case study in chaotic behavior because of good qualitative visualizations and quantitative measurements of chaos, like the Feigenbaum ratio and universality constant τ . Although our experiment studied chaos in the case of an RLD circuit, many of the properties we studied are shared across other chaotic systems, like period doubling and universality. We hope that future studies of chaos in other systems will be informed by our study of chaos in RLD circuits.

7. Acknowledgements

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